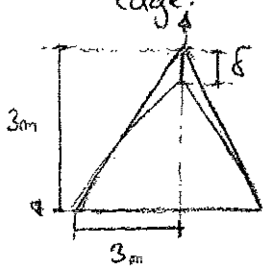
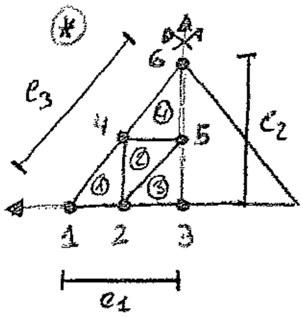
Homework 2.

- Deformed under Self weight.
- Vertical displacement δ on the tip.
- Plane stress
- $t = 1$ (thickness)
- Symmetri only half domain analized (left domain)

1. Strong form of the problem in the reduced domain. Indicate accurately the Boundary Conditions in every edge.





We know that for a equilibrium in a mechanicals problems;

$$\boxed{\nabla \cdot \sigma + \rho b = 0} \quad \text{where;} \quad \nabla \cdot = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) \quad b = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

As our problem is for plane stresses ($\sigma_z = 0$ and $\frac{\partial}{\partial z} = 0$) our system becomes

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \cancel{\frac{\partial}{\partial z}}^{\,0}\right) \cdot \begin{pmatrix} \sigma_x & \tau_{xy} & \cancel{\tau_{xz}}^{\,0} \\ \tau_{yx} & \sigma_y & \cancel{\tau_{yz}}^{\,0} \\ \cancel{\tau_{zx}}^{\,0} & \cancel{\tau_{zy}}^{\,0} & \cancel{\sigma_z}^{\,0} \end{pmatrix} + \rho \cdot \begin{pmatrix} b_x \\ b_y \\ \cancel{b_z}^{\,0} \end{pmatrix};$$

$$; \begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \rho b_x = 0 \\[2ex] \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{yx}}{\partial x} + \rho b_y = 0 \end{cases} ; \quad \text{where;} \quad \boxed{\sigma = D\varepsilon} \text{ and } \boxed{\varepsilon = Ba}$$

we can concluded with the following strong form for our problem;

$$(1) \quad \boxed{B^T \sigma + \rho b = 0}$$

The appropiate Boundary conditions for every edge defined in the ⊛ figure are ; In order to have symetri we put the origin $(0,0)$ in node 3.

$e_1 \rightarrow$ is composed for the nodes $\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ where all are fixed ; e_1 BC $\begin{cases} 1 = (0,0) \\ 2 = (0,0) \\ 3 = (0,0) \end{cases}$

$e_2 \rightarrow$ is composed for the nodes $\begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix}$ where we already know that

3 is fixed : $(0,0)$ as we want keep the symetri of the problem the coordinate x of node 5 and 6 should be fixed as well we know that we have a fixed displacement $-\delta$ on y coordinate of node 6. The only degree of freedom for this edge is y coordinate of node 5. e_2 BC $\begin{cases} 3 = (0,0) \\ 5 = (0,y_5) \\ 6 = (0,-\delta) \end{cases}$

$e_3 \rightarrow$ is composed for the nodes $\begin{Bmatrix} 1 \\ 4 \\ 6 \end{Bmatrix}$ where we already know the BC for 1 and 6. As the node 4 is totally free (no restriccions or displacements applied) we will have 2 degrees of freedom for this edge, the x and y coordinate of node 4. e_3 BC $\begin{cases} 1 = (0,0) \\ 4 = (x_4,y_4) \\ 6 = (0,-\delta) \end{cases}$

2. Describe the mesh shown in figure (*) by giving the nodal coordinates and the conectivity matrix. (For each element the node in the right angle vertex has local number equal to 1).

Element	Local coordinates	Conectivity matrix	Nodal coord.
①	1	2	(-1,5 , 0)
	2	4	(-1,5, 1'5)
	3	1	(-3, 0)
②	1	4	(-1'5 , 1'5)
	2	2	(-1'5, 0)
	3	5	(0, 1,5)
③	1	3	(0 , 0)
	2	5	(0, 1,5)
	3	2	(-1,5, 0)
④	1	5	(0, 1,5)
	2	6	(0, 3)
	3	4	(-1,5, 1,5)

Stiffness matrix of each element:

$$K_e^{(1)} = \begin{array}{c|c} & \begin{array}{ccc} 2 & 4 & 1 \end{array} \\ \begin{array}{c} 2 \\ 4 \\ 1 \end{array} & \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \end{array}, \quad K_e^{(2)} = \begin{array}{c|c} & \begin{array}{ccc} 4 & 2 & 5 \end{array} \\ \begin{array}{c} 4 \\ 2 \\ 5 \end{array} & \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \end{array}, \quad K_e^{(3)} = \begin{array}{c|c} & \begin{array}{ccc} 3 & 5 & 2 \end{array} \\ \begin{array}{c} 3 \\ 5 \\ 2 \end{array} & \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \end{array}$$

$$K_e^{(4)} = \begin{array}{c} \\ 5 \\ 6 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 5 & 6 & 4 \end{array} \\ \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \end{array}$$

; Forces on elements :

$$f_e^{(1)} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \begin{array}{c} 2 \\ 4 \\ 1 \end{array} \; ; \quad f_e^{(2)} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \begin{array}{c} 4 \\ 2 \\ 5 \end{array} \; ; \quad f_e^{(3)} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \begin{array}{c} 3 \\ 5 \\ 2 \end{array} \; ; \quad f_e^{(4)} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \begin{array}{c} 5 \\ 6 \\ 4 \end{array}$$

The finall stiffness matrix looks like :

$$K^G = \begin{array}{c|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \text{lines} \\ \hline 1 & k_{33}^{(1)} & k_{31}^{(1)} & 0 & k_{32}^{(1)} & 0 & 0 & 1\;2 \\ 2 & k_{13}^{(1)} & k_{11}^{(1)}+k_{21}^{(2)}+k_{33}^{(3)} & k_{31}^{(3)} & k_{12}^{(1)}+k_{21}^{(2)} & k_{23}^{(2)}+k_{32}^{(3)} & 0 & 3\;4 \\ 3 & 0 & k_{13}^{(3)} & k_{11}^{(3)} & 0 & k_{12}^{(3)} & 0 & 5\;6 \\ 4 & k_{23}^{(1)} & k_{21}^{(1)}+k_{12}^{(2)} & 0 & k_{22}^{(1)}+k_{11}^{(2)}+k_{33}^{(4)} & k_{13}^{(2)}+k_{31}^{(4)} & k_{32}^{(4)} & 7\;8 \\ 5 & 0 & k_{32}^{(2)}+k_{23}^{(3)} & k_{21}^{(3)} & k_{31}^{(2)}+k_{13}^{(4)} & k_{33}^{(2)}+k_{22}^{(3)}+k_{11}^{(4)} & k_{12}^{(4)} & 9\;10 \\ 6 & 0 & 0 & 0 & k_{23}^{(4)} & k_{21}^{(4)} & k_{22}^{(4)} & 11\;12 \\ \hline & 1\;2 & 3\;4 & 5\;6 & 7\;8 & 9\;10 & 11\;12 & \text{lines} \end{array}$$

= The final force vector looks like:

$$f^G = \begin{bmatrix} f_3^{(1)} \\ f_1^{(1)}+f_2^{(2)}+f_3^{(3)} \\ f_1^{(3)} \\ f_2^{(1)}+f_1^{(2)}+f_3^{(4)} \\ f_3^{(2)}+f_2^{(3)}+f_1^{(4)} \\ f_2^{(4)} \end{bmatrix} \begin{array}{c} 1\;\;1,2 \\ 2\;\;3,4 \\ 3\;\;5,6 \\ 4\;\;7,8 \\ 5\;\;9,10 \\ 6\;\;11,12 \end{array}$$

= The final vector of unknows looks;

applyig BC

$$a^G = \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{2x} \\ a_{2y} \\ a_{3x} \\ a_{3y} \\ a_{4x} \\ a_{4y} \\ a_{5x} \\ a_{5y} \\ a_{6x} \\ a_{6y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ x_4 \\ y_4 \\ 0 \\ y_5 \\ 0 \\ -\delta \end{bmatrix} \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

3. Set up the linear system of equations corresponding to the discretization in figure (*). How many degrees of freedom has the system to be solved?

Starting from the Principle of Virtual Work for 2D.

1
$$\iint_{A^{(e)}} \delta\bar{\varepsilon}^T \bar{\sigma}\, t\, dA = \iint_{A^{(e)}} \delta\bar{u}^T \bar{b}\, t\, dA + \oint_{l^{(e)}} \delta\bar{u}^T \bar{t}\, t\, ds + \sum_{i=1}^{3} \delta u_i^T q_i$$

where; $\delta\bar{\varepsilon} = [\delta\varepsilon_x, \delta\varepsilon_y, \delta\gamma_{xy}]^T$; $\delta\bar{u} = [\delta u, \delta v]^T$; $\bar{b} = [b_x, b_y]^T$; $\bar{t} = [t_x, t_y]$

$q_i = [U_i, V_i]^T$

$\delta u_i = [\delta u_i, \delta v_i]^T$

$t = 1$ and no external forces or surfaces forces are applied in our case. $(\bar{t} = 0 \; ; \; q_i = 0)$

2
$$\iint_{A^{(e)}} \delta\varepsilon\, \sigma\, dA = \iint_{A^{(e)}} \delta\bar{u}\, b\, dA = 0$$

We know that; $\sigma = DBa^{(e)}$ Plane stress.

$\delta u = N\delta a^{(e)} \rightarrow \delta u^T = [\delta a^{(e)}] N^T$

$\delta\varepsilon = B\delta a^{(e)} \rightarrow \delta\varepsilon^T = [\delta a^{(e)}] B^T$

Applied to 2 ;

3
$$\left[\underbrace{\iint_{A^{(e)}} B^T D B a^{(e)} dA}_{K^{(e)} \cdot a^{(e)}} - \underbrace{\iint_{A^{(e)}} N^T b\, dA}_{f_b^{(e)}} = 0 \right]$$

Weak form

$$K^{(e)} \cdot a^{(e)} - f_b^{(e)} = 0$$

$$K^{(e)} = \iint_{A^{(e)}} B^T D B\, dA \; ; \qquad f^{(e)} = \cancel{f_\varepsilon^{(e)}} + \cancel{f_\sigma^{(e)}} + f_b^{(e)} + \cancel{f_t^{(e)}} = \iint_{A^{(e)}} N^T b\, dA.$$

↓

Stiffness matrix of element

In our case $f^{(e)} = f_b^{(e)}$ as we predict before; force on elements are just body forces

We are using triangular elements to discretize the problem then our shape functions will be;

$$N_i = \frac{1}{2A^{(e)}}(a_i + b_i x + c_i y) \quad \text{where;} \left\{ \begin{array}{l} a_i = x_j y_k - x_k y_j \\ b_i = y_j - y_k \\ c_i = x_k - x_j \end{array} \right\} i,j,k = 1,2,3$$

As the strain field is written as $\varepsilon = B a^{(e)}$

where the strain matrix $B = [B_1, B_2, B_3]$ where

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix}$$ for eac node of the element (our case 3 node) $$B = \frac{1}{2A^{(e)}} \left[\begin{array}{cc|cc|cc} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{array} \right]$$

The constitutive matrix D for plane stresses is the following;

$$D = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$; Now we can define the stiffness matrix for our triangular elements.

$$K^{(e)} = \iint_{A^{(e)}} \begin{Bmatrix} B_1^T \\ B_2^T \\ B_3^T \end{Bmatrix} D [B_1, B_2, B_3] \, dA = \iint_{A^{(e)}} \begin{bmatrix} B_1^T D B_1 & B_1^T D B_2 & B_1^T D B_3 \\ & B_2^T D B_2 & B_2^T D B_3 \\ \text{sym.} & & B_3^T D B_3 \end{bmatrix}$$

Finally;

$$\boxed{K_{ij}^{(e)} = \iint_{A^{(e)}} B_i^T D B_j \, dA}$$

The body forces four our problem will look;

$$f_b^{(e)} = \iint_{A^{(e)}} N^T b \, dA = \iint_{A^{(e)}} \begin{bmatrix} N_1^T b \\ N_2^T b \\ N_3^T b \end{bmatrix} dA \quad \text{for each node} \quad f_{b_i}^{(e)} = \iint_{A^{(e)}} N_i^T b \, dA$$

As our body forces is the self weight that can be considered as a uniform force distributed over the element, we obtain;

$$\boxed{f_{b_i} = \frac{A^{(e)}}{3} \begin{Bmatrix} b_x \\ b_y \end{Bmatrix}}$$ where $b_x = 0$ and $b_y = -\rho g$ ρ = density g = gravity.

Finally the unkowns will take the form $a_i^{(e)} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$

where for our problem the final stiffnal matrix and the final vector of forces and unkonws will take the form expressed in the section 2.

K^6, f^6 and a^6. Then taking a look over a^6 with the BC applied we can deduce wihch lines of the final system we will have to solve (how many degrees of freedom he have).

$$a^6 = \begin{array}{c|c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ x_4 \\ y_4 \\ 0 \\ y_5 \\ 0 \\ -\delta \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix} \end{array}$$

Only three degrees of freedom, 3 unkonws to solve in the reduced system

$$a^6_{reduce} = \begin{bmatrix} x_4 \\ y_4 \\ y_5 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 10 \end{matrix}$$

As well the reduce stiffness matrix will look;

$$
\begin{array}{cc}
 & \begin{array}{ccc} 7 & 8 & 10 \end{array} \\
K^6_{red} = \begin{array}{c} 7 \\ 8 \\ 10 \end{array} &
\begin{bmatrix}
K1_{3,3} + K2_{1,1} + K4_{5,5} & K1_{3,4} + K2_{1,2} + K4_{5,6} & K2_{1,6} + K4_{5,2} \\
K1_{4,3} + K2_{2,1} & K1_{4,4} + K2_{2,2} + K4_{6,6} & K2_{2,6} + K4_{6,2} \\
K2_{6,1} + K4_{2,5} & K2_{6,2} + K4_{2,6} & K2_{6,6} + K3_{4,4} + K4_{2,2}
\end{bmatrix}
\end{array}
$$

lines

and the reduce vector of forces;

$$
fb^6_{reduce} = \begin{bmatrix} 3 \cdot fb_{11} \\ 3 \cdot fb_{21} \\ 3 \cdot fb_{21} \end{bmatrix} \begin{array}{c} 7 \\ 8 \\ 10 \end{array}
$$

lines

Final system, to solve

$$
\boxed{K^6_{reduce} \cdot a^6_{reduce} = fb^6_{reduce}}
$$

4. Compute the FE approximation u^h.

Use $E = 10\,GPa$, $\nu = 0.2$, $\delta = 10^{-2}\,m$, $\rho g = 10^3\,N/m^2$

where $A^{(e)} = \frac{1.5 \cdot 1.5}{2} = 9/8$

To compute the solution we use a code programed in Maple that calculate and solves the reduced final system described before.

: Results;

$$
\begin{cases}
x_4 = -0{,}0001282051280 \\
y_4 = -0{,}001132586632 \\
y_5 = -0{,}003867629368
\end{cases}
$$

(*) code attached.

Maple Code used to solve the section 4 :

```
> with(LinearAlgebra):
> pg := 10^3;
  t := 1;
  A := 9/8;
  E := 10*10^9;
  v := 0.2;
  delt := 10^(-2);

> d := (E/(1 - v^2)) . Matrix([[1, v, 0], [v, 1, 0], [0, 0, (1 - v)/2]]);

  b11 := 1.5;
  c11 := -1.5;
  b21 := 0;
  c21 := 1.5;
  b31 := -1.5;
  c31 := 0;
  B1 := (1/(2*A)) . Matrix([[b11, 0, b21, 0, b31, 0], [0, c11, 0, c21, 0, c31], [c11, b11, c21, b21, c31, b31]]);
  k1 := (Transpose(B1).d.B1)*A;
  b12 := -1.5;
  c12 := 1.5;
  b22 := 0;
  c22 := -1.5;
  b32 := 1.5;
  c32 := 0;
  B2 := (1/(2*A)) . Matrix([[b12, 0, b22, 0, b32, 0], [0, c12, 0, c22, 0, c32], [c12, b12, c22, b22, c32, b32]]);
  k2 := (Transpose(B2).d.B2)*A;
  b13 := 1.5;
  c13 := -1.5;
  b23 := 0;
  c23 := 1.5;
  b33 := -1.5;
  c33 := 0;
```

$$B3 := \frac{1}{2 \cdot A} \cdot \begin{bmatrix} b13 & 0 & b23 & 0 & b33 & 0 \\ 0 & c13 & 0 & c23 & 0 & c33 \\ c13 & b13 & c23 & b23 & c33 & b33 \end{bmatrix};$$

$k3 := (Transpose(B3).d.B3) \cdot A;$
$b14 := 1.5;$
$c14 := -1.5;$
$b24 := 0;$
$c24 := 1.5;$
$b34 := -1.5;$
$c34 := 0;$

$$B4 := \left(\frac{1}{2 \cdot A}\right) \cdot \begin{bmatrix} b14 & 0 & b24 & 0 & b34 & 0 \\ 0 & c14 & 0 & c24 & 0 & c34 \\ c14 & b14 & c24 & b24 & c34 & b34 \end{bmatrix};$$

$k4 := (Transpose(B4).d.B4) \cdot A;$

$$> fb := \left(\frac{A \cdot t}{3}\right) \cdot \begin{bmatrix} 0 \\ -pg \end{bmatrix};$$

$$> Kfinal := \begin{bmatrix} k1_{3,3} + k2_{1,1} + k4_{5,5} & k1_{3,4} + k2_{1,2} + k4_{5,6} & k2_{1,6} + k4_{5,2} & k4_{5,4} \\ k1_{4,3} + k2_{2,1} + k4_{6,5} & k1_{4,4} + k2_{2,2} + k4_{6,6} & k2_{2,6} + k4_{6,2} & k4_{6,4} \\ k2_{6,1} + k4_{2,5} & k2_{6,2} + k4_{2,6} & k2_{6,6} + k3_{4,4} + k4_{2,2} & k4_{2,4} \end{bmatrix};$$

$$a := \begin{bmatrix} x4 \\ y4 \\ y5 \\ -delt \end{bmatrix};$$

$$ff := \begin{bmatrix} 3 \cdot 0 \\ -375 \cdot 3 \\ -375 \cdot 3 \end{bmatrix};$$

$> Finalsyst := Kfinal.a = ff;$
$Finalsyst(1);$
$Finalsyst(2);$
$Finalsyst(3);$

$> solve(\{ Finalsyst(1), Finalsyst(2), Finalsyst(3) \}, [x4, y4, y5]);$

Results:

Initial data:

$$pg := 1000$$

$$t := 1$$

$$A := \frac{9}{8}$$

$$E := 10000000000$$

$$\nu := 0.2$$

$$delt := \frac{1}{100}$$

Constitutive Matrix (Plane Stresses):

$$d := \begin{bmatrix} 1.041666667\ 10^{10} & 2.083333334\ 10^{9} & 0. \\ 2.083333334\ 10^{9} & 1.041666667\ 10^{10} & 0. \\ 0. & 0. & 4.166666668\ 10^{9} \end{bmatrix}$$

Strain matrix and Stiffness matrix for each element:

1.Element:

$$b11 := 1.5$$

$$c11 := -1.5$$

$$b21 := 0$$

$$c21 := 1.5$$

$$b31 := -1.5$$

$$c31 := 0$$

$$B1 := \begin{bmatrix} 0.666666666499999950 & 0. & 0. & 0. & -0.666666666499999950 & 0. \\ 0. & -0.666666666499999950 & 0. & 0.666666666499999950 & 0. & 0. \\ -0.666666666499999950 & 0.666666666499999950 & 0.666666666499999950 & 0. & 0. & -0.666666666499999950 \end{bmatrix}$$

$$k1 :=$$

$$\begin{bmatrix} 7.29166666535416508\ 10^{9} & -3.12499999943749953\ 10^{9} & -2.08333333295833302\ 10^{9} & 1.04166666647916651\ 10^{9} & -5.20833333239583302\ 10^{9} & 2.08333333295833302\ 10^{9} \\ -3.12499999943749953\ 10^{9} & 7.29166666535416508\ 10^{9} & 2.08333333295833302\ 10^{9} & -5.20833333239583302\ 10^{9} & 1.04166666647916651\ 10^{9} & -2.08333333295833302\ 10^{9} \\ -2.08333333295833302\ 10^{9} & 2.08333333295833302\ 10^{9} & 2.08333333295833302\ 10^{9} & 0. & 0. & -2.08333333295833302\ 10^{9} \\ 1.04166666647916651\ 10^{9} & -5.20833333239583302\ 10^{9} & 0. & 5.20833333239583302\ 10^{9} & -1.04166666647916651\ 10^{9} & 0. \\ -5.20833333239583302\ 10^{9} & 1.04166666647916651\ 10^{9} & 0. & -1.04166666647916651\ 10^{9} & 5.20833333239583302\ 10^{9} & 0. \\ 2.08333333295833302\ 10^{9} & -2.08333333295833302\ 10^{9} & -2.08333333295833302\ 10^{9} & 0. & 0. & 2.08333333295833302\ 10^{9} \end{bmatrix}$$

2. Element:

$$b12 := -1.5$$
$$c12 := 1.5$$
$$b22 := 0$$
$$c22 := -1.5$$
$$b32 := 1.5$$
$$c32 := 0$$

$$B2 := \begin{bmatrix} -0.666666666499999950 & 0. & 0. & 0. & 0.666666666499999950 & 0. \\ 0. & 0.666666666499999950 & 0. & -0.666666666499999950 & 0. & 0. \\ 0.666666666499999950 & -0.666666666499999950 & -0.666666666499999950 & 0. & 0. & 0.666666666499999950 \end{bmatrix}$$

$k2 :=$

$$\begin{bmatrix} 7.29166666535416508\ 10^9 & -3.12499999943749953\ 10^9 & -2.08333333295833302\ 10^9 & 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 2.08333333295833302\ 10^9 \\ -3.12499999943749953\ 10^9 & 7.29166666535416508\ 10^9 & 2.08333333295833302\ 10^9 & -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & -2.08333333295833302\ 10^9 \\ -2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 0. & 0. & -2.08333333295833302\ 10^9 \\ 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 0. & 5.20833333239583302\ 10^9 & -1.04166666647916651\ 10^9 & 0. \\ -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & 0. & -1.04166666647916651\ 10^9 & 5.20833333239583302\ 10^9 & 0. \\ 2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & 0. & 0. & 2.08333333295833302\ 10^9 \end{bmatrix}$$

3. Element:

$$b13 := 1.5$$
$$c13 := -1.5$$
$$b23 := 0$$
$$c23 := 1.5$$
$$b33 := -1.5$$
$$c33 := 0$$

$$B3 := \begin{bmatrix} 0.666666666499999950 & 0. & 0. & 0. & -0.666666666499999950 & 0. \\ 0. & -0.666666666499999950 & 0. & 0.666666666499999950 & 0. & 0. \\ -0.666666666499999950 & 0.666666666499999950 & 0.666666666499999950 & 0. & 0. & -0.666666666499999950 \end{bmatrix}$$

$k3 :=$

$$\begin{bmatrix} 7.29166666535416508\ 10^9 & -3.12499999943749953\ 10^9 & -2.08333333295833302\ 10^9 & 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 2.08333333295833302\ 10^9 \\ -3.12499999943749953\ 10^9 & 7.29166666535416508\ 10^9 & 2.08333333295833302\ 10^9 & -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & -2.08333333295833302\ 10^9 \\ -2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 0. & 0. & -2.08333333295833302\ 10^9 \\ 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 0. & 5.20833333239583302\ 10^9 & -1.04166666647916651\ 10^9 & 0. \\ -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & 0. & -1.04166666647916651\ 10^9 & 5.20833333239583302\ 10^9 & 0. \\ 2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & 0. & 0. & 2.08333333295833302\ 10^9 \end{bmatrix}$$

4. Element:

$$b14 := 1.5$$
$$c14 := -1.5$$
$$b24 := 0$$
$$c24 := 1.5$$
$$b34 := -1.5$$
$$c34 := 0$$

$$B4 := \begin{bmatrix} 0.666666666499999950 & 0. & 0. & 0. & -0.666666666499999950 & 0. \\ 0. & -0.666666666499999950 & 0. & 0.666666666499999950 & 0. & 0. \\ -0.666666666499999950 & 0.666666666499999950 & 0.666666666499999950 & 0. & 0. & -0.666666666499999950 \end{bmatrix}$$

$k4 :=$

$$\begin{bmatrix} 7.29166666535416508\ 10^9 & -3.12499999943749953\ 10^9 & -2.08333333295833302\ 10^9 & 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 2.08333333295833302\ 10^9 \\ -3.12499999943749953\ 10^9 & 7.29166666535416508\ 10^9 & 2.08333333295833302\ 10^9 & -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & -2.08333333295833302\ 10^9 \\ -2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 2.08333333295833302\ 10^9 & 0. & 0. & -2.08333333295833302\ 10^9 \\ 1.04166666647916651\ 10^9 & -5.20833333239583302\ 10^9 & 0. & 5.20833333239583302\ 10^9 & -1.04166666647916651\ 10^9 & 0. \\ -5.20833333239583302\ 10^9 & 1.04166666647916651\ 10^9 & 0. & -1.04166666647916651\ 10^9 & 5.20833333239583302\ 10^9 & 0. \\ 2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & -2.08333333295833302\ 10^9 & 0. & 0. & 2.08333333295833302\ 10^9 \end{bmatrix}$$

Body forces:

$$fb := \begin{bmatrix} 0 \\ -375 \end{bmatrix}$$

Final reduced system:

$$Finalsyst := \begin{bmatrix} 1.458333333\ 10^{10}\ x4 - 3.124999999\ 10^{9}\ y4 + 3.124999999\ 10^{9}\ y5 + 1.041666666\ 10^{7} \\ -3.124999999\ 10^{9}\ x4 + 1.458333333\ 10^{10}\ y4 - 4.166666666\ 10^{9}\ y5 \\ 3.124999999\ 10^{9}\ x4 - 4.166666666\ 10^{9}\ y4 + 1.458333333\ 10^{10}\ y5 + 5.208333332\ 10^{7} \end{bmatrix} = \begin{bmatrix} 0 \\ -1125 \\ -1125 \end{bmatrix}$$

Solutions:

$$[[x4 = -0.0001282051280, y4 = -0.001132586632, y5 = -0.003867629368]]$$