FINITE ELEMENTS

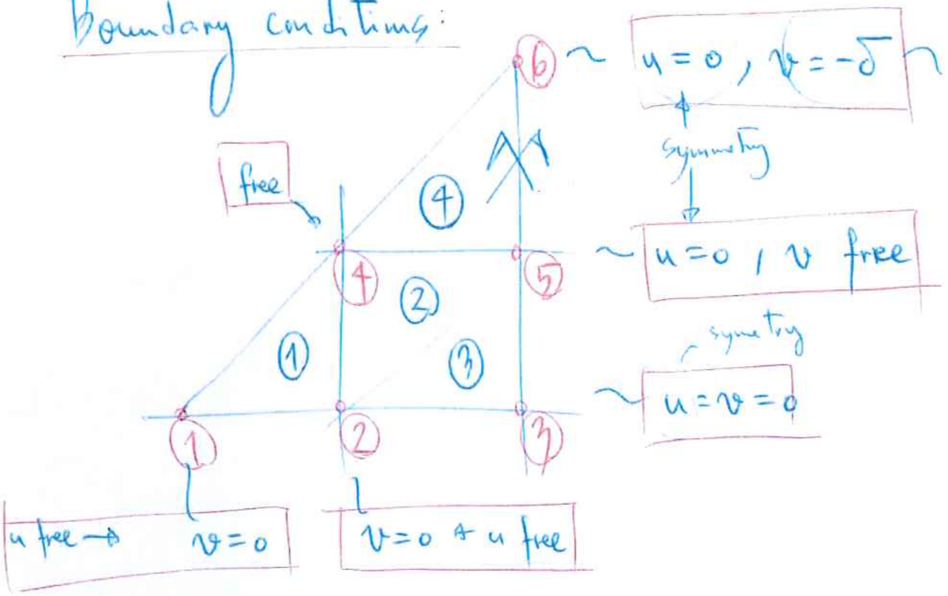
HOMEWORK 2
Plane Elasticity

1. Plane stress $\Rightarrow \boxed{\sigma_z = \gamma_{xz} = \gamma_{yz} = 0}$

$\underline{\sigma} = \underline{\underline{D}} \cdot \underline{\varepsilon}$, where
$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau_{xy} \end{pmatrix}$$

And $\boxed{d_{11} = d_{22} = \dfrac{E}{1-\nu^2}}$; $\boxed{d_{12} = d_{21} = \dfrac{\nu \cdot E}{1-\nu^2}}$; $\boxed{d_{33} = \dfrac{E}{2(1+\nu)} = G}$

Boundary conditions:



2.

Node	Coordinates
1	(0, 0)
2	(1.5, 0)
3	(3, 0)
4	(1.5, 1.5)
5	(3, 1.5)
6	(3, 3)

Element	Nodal connections			
	1	2	3	local numbering
1	2	4	1	
2	4	2	5	global numbering
3	3	5	2	
4	5	6	4	

3.

reactions at the bottom

$$
\begin{pmatrix}
k_{33}^1 & k_{13}^1 & 0 & k_{23}^1 & 0 & 0 \\
 & k_{11}^1+k_{22}^2+k_{33}^3 & k_{13}^3 & k_{12}^1+k_{12}^2 & k_{23}^2+k_{23}^3 & 0 \\
 & & k_{11}^3 & 0 & k_{12}^3 & 0 \\
 & & & k_{22}^1+k_{11}^2+k_{33}^4 & k_{13}^2+k_{14}^4 & k_{23}^4 \\
 & \text{SYMM.} & & & k_{33}^2+k_{11}^4+k_{22}^3 & k_{12}^4 \\
 & & & & & k_{22}^4
\end{pmatrix}
\cdot
\begin{pmatrix}
\bar{a}_1 \\ \bar{a}_2 \\ \bar{a}_3 \\ \bar{a}_4 \\ \bar{a}_5 \\ \bar{a}_6
\end{pmatrix}
=
\underbrace{\begin{pmatrix}
\bar{r}_1 + f_3^1 \\
r_2 + f_1^1 + f_2^2 + f_3^3 \\
r_3 + f_1^3 \\
f_2^1 + f_1^2 + f_3^4 \\
f_3^2 + f_2^3 + f_1^4 \\
f_2^4
\end{pmatrix}}_{\text{surface density should be added at nodes}}
$$

Initially, the system has 12 unknowns. Nevertheless, the ~~6~~ 7 boundary conditions leaves the problem with only 5 unknowns.

4. We must set up the standard system of equations $\underset{\sim}{K}\cdot \underline{a} = \underline{f}$

displacements

where $\underset{\sim}{K}_e^{ij} = \int_A \underset{\sim}{B}^T \cdot \underset{\sim}{D} \cdot \underset{\sim}{B} \cdot t \cdot dA$

~1m

constitutive matrix

area of triangle — deformation matrix

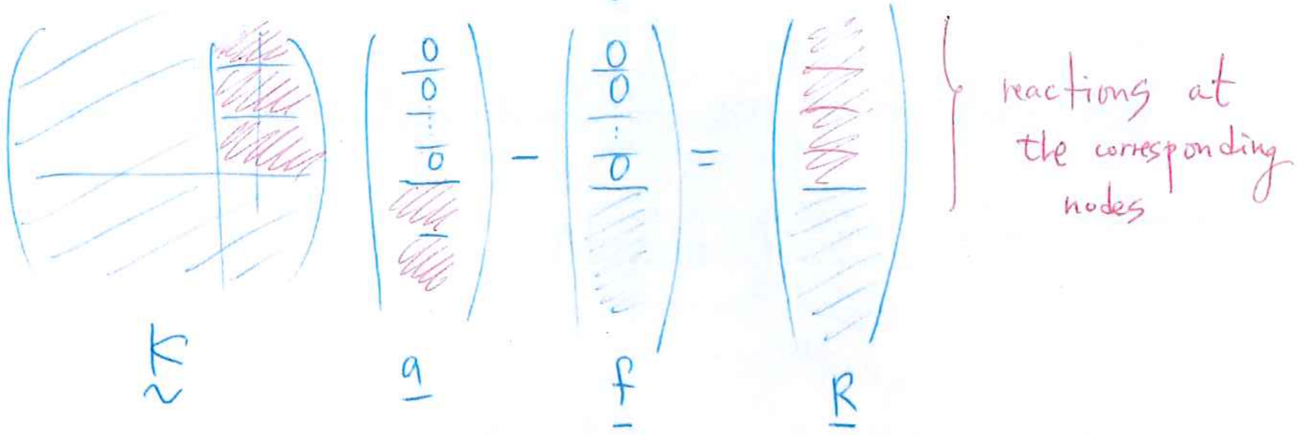
and $\underline{f}_e^i = \int_A \underline{N}_i^T \cdot \underline{b} \cdot t \cdot dA = \dfrac{A\,t}{3}\begin{pmatrix} 0 \\ -\rho g \end{pmatrix}$

thickness — surface gravity — gravity

After calculating $\underset{\sim}{K}$, we obtain the following structure:

$$\underset{\sim}{K} = \left(\begin{array}{c|c|c|c}
\begin{matrix} 5\cdot10^9 & -5\cdot10^9 & \\ & 1 & \\ & 1.5\cdot10^{10} & \end{matrix} & 0 & \begin{matrix} & -1\cdot10^9 \\ -4\cdot10^9 & 3\cdot10^9 \end{matrix} & \begin{matrix} \\ -3\cdot10^9 \end{matrix} \\ \hline
0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 1 \end{matrix} & 0 & 0 \\ \hline
0 & 0 & \begin{matrix} 1.5\cdot10^{10} & -3\cdot10^9 & \\ & 1.5\cdot10^{10} & \\ & & 1 \end{matrix} & \begin{matrix} 3\cdot10^9 & -1\cdot10^9 \\ -4\cdot10^9 & \end{matrix} \\ \hline
0 & 0 & 0 & \begin{matrix} 1.5\cdot10^{10} & & 5\cdot10^9 \\ & 1 & \\ & & 5\cdot10^9 \end{matrix}
\end{array}\right) ; \quad \underline{f} = \begin{pmatrix} \\ \\ \\ 0 \\ -112.5 \\ 0 \\ -138.9 \\ 0 \\ -15.5 \end{pmatrix}$$

To properly calculate $\underline{f}$ and reactions $\underline{R}$, we just follow the following methodology: (skematic)



After fully solving the system, we obtain the solution $\bar{u}^h$:
In meters.